

Lectures on Supersymmetry

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Classroom-Style Lecture Series, Fermilab, Batavia, IL, June 23 and 30, 2005.

Standard Model Particles

There are 12 fundamental gauge fields:

8 gluons, 3 W_μ 's and B_μ

and 3 gauge couplings g_1 , g_2 , g_3

The matter fields:

3 families of quarks and leptons with same quantum numbers under gauge groups

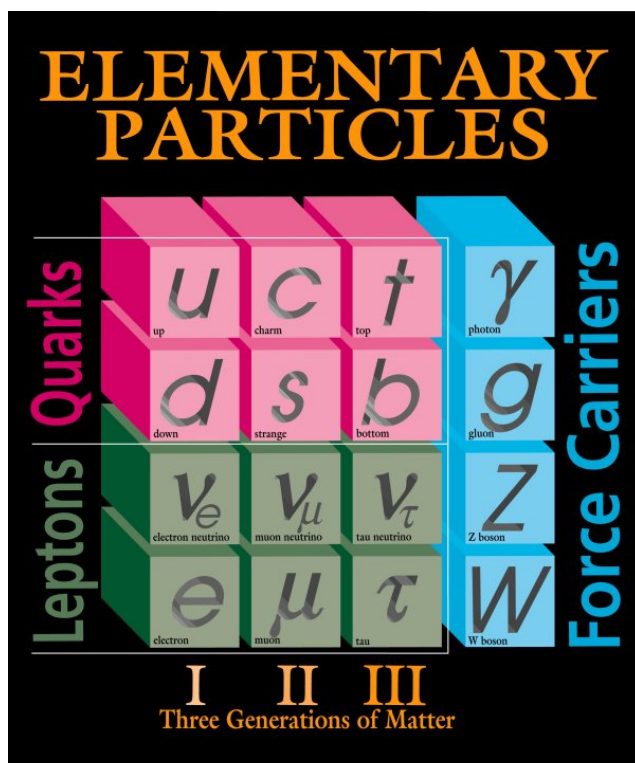
But very different masses!

m_3/m_2 and $m_2/m_1 \simeq$ a few tens or hundreds
 $m_e = 0.5 \cdot 10^{-3} \text{ GeV}$, $\frac{m_\mu}{m_e} \simeq 200$, $\frac{m_\tau}{m_\mu} \simeq 20$

Largest hierarchies

$m_t \simeq 175 \text{ GeV}$ $m_t/m_e \propto 10^5$

neutrino masses smaller than as 10^{-9} GeV !



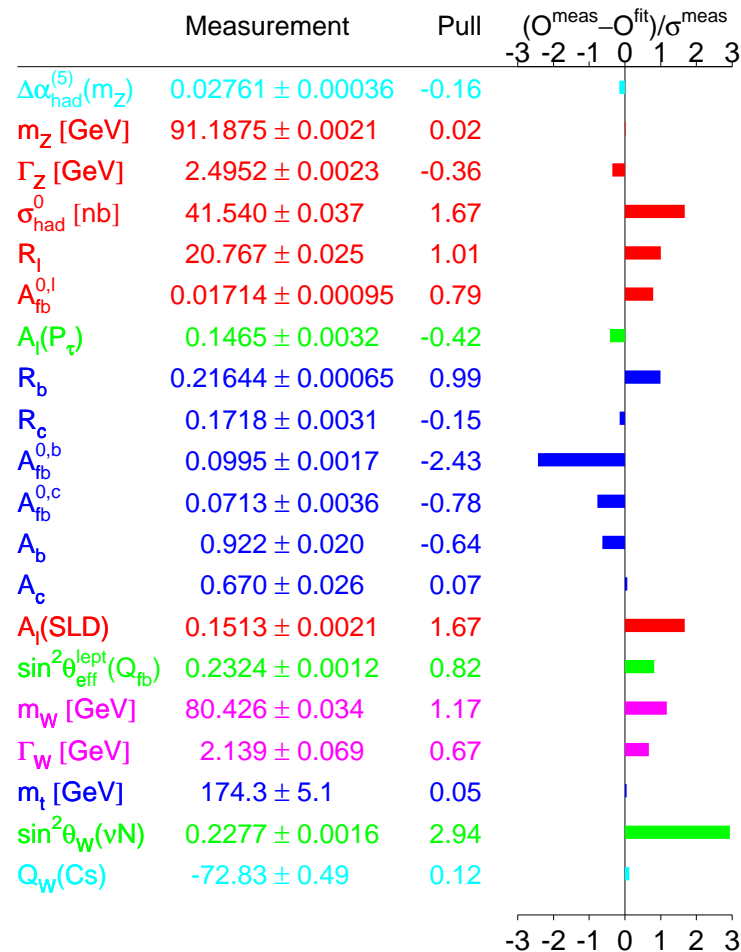
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FERMIONS			matter constituents spin = 1/2, 3/2, 5/2, ...		
Leptons spin = 1/2			Quarks spin = 1/2		
Flavor	Mass GeV/c ²	Electric charge	Flavor	Approx. Mass GeV/c ²	Electric charge
ν_e electron neutrino	$<1 \times 10^{-8}$	0	u up	0.003	2/3
e electron	0.000511	-1	d down	0.006	-1/3
ν_μ muon neutrino	<0.0002	0	c charm	1.3	2/3
μ muon	0.106	-1	s strange	0.1	-1/3
ν_τ tau neutrino	<0.02	0	t top	175	2/3
τ tau	1.7771	-1	b bottom	4.3	-1/3

Precision Tests of the SM

- The SM has been tested with very high precision (one part in a thousand) at experiments around the world: CERN, Fermilab, SLAC

Winter 2003



Spontaneous Symmetry Breakdown

Particle Masses arise through the Higgs mechanism: Spontaneous breakdown of gauge symmetry

$$SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{\text{em}} \quad (1)$$

A scalar field, charged under the gauge group, acquires v.e.v.

$$V(H) = m_H^2 H^\dagger H + \frac{\lambda}{2} (H^\dagger H)^2 \quad (2)$$

Therefore,

$$\langle H^\dagger H \rangle = -\frac{m_H^2}{\lambda} \quad (3)$$

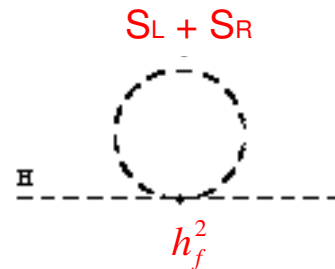
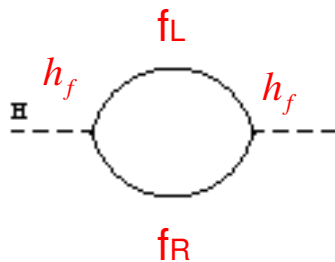
the v.e.v. of the Higgs field is fixed by the value of the negative mass parameter.

Problem: The mass parameter is unstable under quantum corrections.

Higgs Mass Parameter Corrections

One loop corrections to the Higgs mass parameter cancel if the couplings of scalars and fermions are equal to each other

$$\delta m_H^2 = \frac{N_c h_f^2}{16\pi^2} \left[-2\Lambda^2 + 3m_f^2 \log\left(\frac{\Lambda^2}{m_f^2}\right) + 2\Lambda^2 - 2m_s^2 \log\left(\frac{\Lambda^2}{m_s^2}\right) \right]$$



(If the masses proceed from the v.e.v. of H, there is another diagram that ensures also the cancellation of the log term. Observe that the fermion and scalar masses are the same in this case, equal to $h_f v$.)

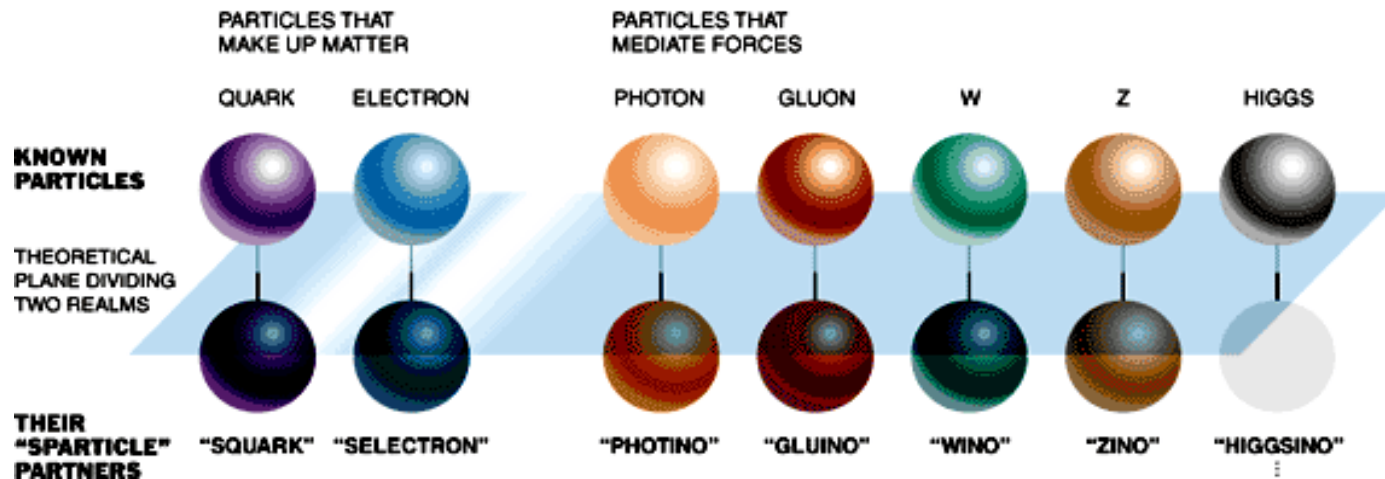
Supersymmetry is a symmetry that ensures the equality of these couplings.

supersymmetry

fermions



bosons



Photino, Zino and Neutral Higgsino: Neutralinos

Charged Wino, charged Higgsino: Charginos

No new dimensionless couplings. Couplings of supersymmetric particles equal to couplings of Standard Model ones.

Two Higgs doublets necessary. Ratio of vacuum expectation values denoted by $\tan \beta$

Why Supersymmetry ?

- Helps to stabilize the weak scale—Planck scale hierarchy
- Supersymmetry algebra contains the generator of space-time translations.
Necessary ingredient of theory of quantum gravity.
- Minimal supersymmetric extension of the SM :
Leads to Unification of gauge couplings
- Starting from positive masses at high energies, electroweak symmetry breaking is induced radiatively
- If discrete symmetry, $P = (-1)^{3B+L+2S}$ is imposed, lightest SUSY particle neutral and stable: Excellent candidate for cold Dark Matter.

Structure of Supersymmetric Gauge Theories

- The Standard Model is based on a Gauge Theory.
- A supersymmetric extension of the Standard Model has then to follow the rules of Supersymmetric Gauge Theories.
- These theories are based on two set of fields:
 - Chiral fields, that contain left handed components of the fermion fields and their superpartners.
 - Vector fields, containing the vector gauge bosons and their superpartners.
- Right-handed fermions are contained on chiral fields by means of their charge conjugate representation

$$(\psi_R)^C = (\psi^C)_L \quad (4)$$

- Higgs fields are described by chiral fields, with fermion superpartners

Generators of Supersymmetry

- Supersymmetry is a symmetry that relates boson to fermion degrees of freedom, $Q|F\rangle = |B\rangle$, $Q|B\rangle = |F\rangle$.
- The generators of supersymmetry are two component anticommuting spinors, Q_α , $\bar{Q}^{\dot{\alpha}}$, satisfying

$$\{Q_\alpha, Q_\beta\} = 0 \quad (5)$$

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma_{\alpha\dot{\beta}}^\mu P_\mu \quad (6)$$

where $\sigma^\mu = (I, \vec{\sigma})$, $\bar{\sigma}^\mu = (I, -\vec{\sigma})$, and σ^i are the Pauli matrices. As anticipated, space-time translations are part of the SUSY algebra.

- Two-spinors may be contracted to form Lorentz invariant quantities

$$\psi^\alpha \chi_\alpha = \psi^\alpha \epsilon_{\alpha\beta} \chi^\beta \quad (7)$$

Four-component vs. Two-component fermions

- A Dirac Spinor is a four component object whose components are

$$\psi_D = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}; \quad \psi_D^C = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} \quad (8)$$

- A Majorana Spinor is a four component object whose components are

$$\psi_M = \begin{pmatrix} \chi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}; \quad \psi_M^C = \psi_M \quad (9)$$

- Gamma Matrices

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}; \quad \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix} \quad (10)$$

- Observe that $\psi_{D,L} = \chi$; $\psi_{D,R} = \bar{\psi}$

- Usual Dirac contractions may be then expressed in terms of two component contractions.

$$\bar{\psi}_D = (\psi^\alpha \quad \bar{\chi}_{\dot{\alpha}}) \quad (11)$$

- For instance,

$$\bar{\psi}_D \psi_D = \psi\chi + h.c.; \quad (12)$$

$$\bar{\psi}_D \gamma^\mu \psi_D = \psi \bar{\sigma}^\mu \bar{\psi} + \bar{\chi} \sigma^\mu \chi = -\bar{\psi} \sigma^\mu \psi + \bar{\chi} \sigma^\mu \chi \quad (13)$$

Observe that Majorana particles lead to vanishing vector currents. Therefore, they must be neutral under electromagnetic interactions. Chiral currents don't vanish, $\bar{\psi}_D \gamma^\mu \gamma_5 \psi_D = -\bar{\psi} \sigma^\mu \psi - \bar{\chi} \sigma^\mu \chi$. They may couple to the Z -boson.

- Other relations may be found in the literature.

Superspace

- In order to describe supersymmetric theories, it proves convenient to introduce the concept of superspace.
- Apart from the ordinary coordinates x^μ , one introduces new anticommuting spinor coordinates θ^α and $\bar{\theta}_{\dot{\alpha}}$; $[\theta] = [\bar{\theta}] = -1/2$.
- This allows to represent fermion and boson fields by the same superfield.
- For instance, a generic chiral field, forming the base for an irreducible representation of SUSY, is given by

$$\Phi(x, \theta, \bar{\theta} = 0) = A(x) + \sqrt{2} \theta \psi(x) + \theta^2 F(x) \quad (14)$$

$$\Phi(x, \theta, \bar{\theta}) = \exp(-i\partial_\mu \theta \sigma^\mu \bar{\theta}) \Phi(x, \theta, \bar{\theta} = 0) \quad (15)$$

- A , ψ and F are the scalar, fermion and auxiliary components.

Transformation of chiral field components

- Supersymmetry is a particular translation in superspace, characterized by a Grassman parameter ξ .
- Supersymmetry generators may be given as derivative operators

$$Q_\alpha = i \left[-\partial_\theta - i\sigma^\mu \bar{\theta} \partial_\mu \right] \quad (16)$$

- Under supersymmetric transformations, the components of chiral fields transform like

$$\begin{aligned} \delta A &= \sqrt{2}\xi\psi, & \delta F &= -i\sqrt{2}\bar{\xi}\bar{\sigma}^\mu\partial_\mu\psi \\ \delta\psi &= -i\sqrt{2}\sigma^\mu\bar{\xi}\partial_\mu A + \sqrt{2}\xi F \end{aligned} \quad (17)$$

- Interestingly enough, the F component transforms like a total derivative and it is a good guidance to construct supersymmetric Lagrangians.

Properties of chiral superfields

- The product of two superfields is another superfield.
- For instance, the F-component of the product of two superfields Φ_1 and Φ_2 is obtained by collecting all the terms in θ^2 , and is equal to

$$A_1 F_2 + A_2 F_1 + \psi_1 \psi_2 \quad (18)$$

- For a generic Polynomial function of several fields $P(\Phi_i)$, the result is

$$(\partial_{A_i} P(A)) F_i + \frac{1}{2} \left(\partial_{A_i, A_j}^2 P(A) \right) \psi_i \psi_j \quad (19)$$

- Finally, a single chiral field has dimensionality $[A] = [\Phi] = 1$, $[\psi] = 3/2$ and $[F] = 2$. For $P(A)$, $[P(\Phi)]_F = [P(\Phi)] + 1$ ($[\theta] = [\bar{\theta}] = -1/2$).

Vector Superfields

- Vector Superfields are generic hermitian fields. The minimal irreducible representations may be obtained by

$$V(x, \theta, \bar{\theta}) = - (\theta \sigma^\mu \bar{\theta}) V_\mu + i \theta^2 \bar{\theta} \bar{\lambda} - i \bar{\theta}^2 \theta \lambda + \frac{1}{2} \theta^2 \bar{\theta}^2 D \quad (20)$$

- Vector Superfields contain a regular vector field V_μ , its fermionic supersymmetric partner λ and an auxiliary scalar field D .
- The D-component of a vector field transform like a total derivative.
- $D = [V] + 2$; $[V_\mu] = [V] + 1$; $[\lambda] = [V] + 3/2$. If V_μ describes a physical gauge field, then $[V] = 0$.

Superfield Strength and gauge transformations

- Similarly to $F_{\mu\nu}$ in the regular case, there is a field that contains the field strength. It is a chiral field, derived from V , and it is given by

$$W^\alpha(x, \theta, \bar{\theta} = 0) = -i\lambda^\alpha + (\theta\sigma_{\mu\nu})^\alpha F^{\mu\nu} + \theta^\alpha D - \theta^2 (\bar{\sigma}^\mu \mathcal{D}_\mu \bar{\lambda})^\alpha \quad (21)$$

- Under gauge transformations, superfields transform like

$$\begin{aligned} \Phi &\rightarrow \exp(-ig\Lambda)\Phi, & W_\alpha &\rightarrow \exp(-ig\Lambda)W_\alpha \exp(ig\Lambda) \\ \exp(gV) &\rightarrow \exp(-ig\bar{\Lambda}) \exp(gV) \exp(ig\Lambda) \end{aligned} \quad (22)$$

where Λ is a chiral field of dimension 0.

Towards a Supersymmetric Lagrangian

- The aim is to construct a Lagrangian, invariant under supersymmetry and under gauge transformations.
- One should remember, for that purpose, that both the F-component of a chiral field, as well as the D-component of a vector field transform under SUSY as a total derivative.
- One should also remember that, if renormalizability is imposed, then the dimension of all interaction terms in the Lagrangian

$$[\mathcal{L}_{\text{int}}] \leq 4 \quad (23)$$

- On the other hand,

$$[\Phi] = 1, \quad [W_\alpha] = 3/2, \quad [V] = 0. \quad (24)$$

and one should remember that $[V]_D = [V] + 2$; $[\Phi]_F = [\Phi] + 1$.

Supersymmetric Lagrangian

- Once the above machinery is introduced, the total Lagrangian takes a particular simple form. The total Lagrangian is given by

$$\begin{aligned}\mathcal{L}_{\text{SUSY}} = & \frac{1}{4g^2} (Tr[W^\alpha W_\alpha]_F + h.c.) + \sum_i (\bar{\Phi} \exp(gV) \Phi)_D \\ & + ([P(\Phi)]_F + h.c.)\end{aligned}\tag{25}$$

where $P(\Phi)$ is the most generic **dimension-three, gauge invariant**, polynomial function of the chiral fields Φ , and it is called **Superpotential**. It has the general expression

$$P(\Phi) = c_i \Phi_i + \frac{m_{ij}}{2} \Phi_i \Phi_j + \frac{\lambda_{ijk}}{3!} \Phi_i \Phi_j \Phi_k\tag{26}$$

- The D-terms of V^a and the F term of Φ_i do not receive any derivative contribution: Auxiliary fields that can be integrated out by equation of motion.

Lagrangian in terms of Component Fields

- The above Lagrangian has the usual kinetic terms for the boson and fermion fields. It also contain generalized Yukawa interactions and contain interactions between the gauginos, the scalar and the fermion components of the chiral superfields.

$$\begin{aligned}\mathcal{L}_{\text{SUSY}} = & (\mathcal{D}_\mu A_i)^\dagger \mathcal{D} A_i + \left(\frac{i}{2} \bar{\psi}_i \bar{\sigma}^\mu \mathcal{D}_\mu \psi_i + \text{h.c.} \right) \\ & - \frac{1}{4} (G_{\mu\nu}^a)^2 + \left(\frac{i}{2} \bar{\lambda}^a \bar{\sigma}^\mu \mathcal{D}_\mu \lambda^a + \text{h.c.} \right) \\ & - \left(\frac{1}{2} \frac{\partial^2 P(A)}{\partial A_i \partial A_j} \psi_i \psi_j - i\sqrt{2} g A_i^* T_a \psi_i \lambda^a + \text{h.c.} \right) \\ & - V(F_i, F_i^*, D^a)\end{aligned}\tag{27}$$

- The last term is a potential term that depend only on the auxiliary fields

Notation Refreshment

- All **standard matter fermion fields** are described by their left-handed components (using the charge conjugates for right-handed fields) ψ_i
- All standard matter **fermion superpartners** are described the scalar fields A_i . There is one for each chiral fermion.
- **Gauge bosons** are inside covariant derivatives and in the $G_{\mu\nu}$ terms.
- **Gauginos**, the superpartners of the gauge bosons are described by the fermion fields λ_a . There is one Weyl fermion for each massless gauge boson.
- **Higgs bosons** and their superpartners are described as **regular chiral fields**. Their only distinction is that their scalar components acquire a v.e.v. and, as we will see, they are the only scalars with positive R-Parity.

Scalar Potential

$$V(F_i, F_i^*, D^a) = \sum_i F_i^* F_i + \frac{1}{2} \sum_a (D^a)^2 \quad (28)$$

where the auxiliary fields may be obtained from their equation of motion, as a function of the scalar components of the chiral fields:

$$F_i^* = -\frac{\partial P(A)}{\partial A_i}, \quad D^a = -g \sum_i (A_i^* T^a A_i) \quad (29)$$

Observe that the **quartic couplings** are governed by the **gauge couplings** and that scalar potential is positive definite ! The latter is not a surprise. From the supersymmetry algebra, one obtains,

$$H = \frac{1}{4} \sum_{\alpha=1}^2 (Q_\alpha^\dagger Q_\alpha + Q_\alpha Q_\alpha^\dagger) \quad (30)$$

- If for a physical state the energy is zero, this is the ground state.
- Supersymmetry is broken if the vacuum energy is non-zero !

Couplings

- The Yukawa couplings between scalar and fermion fields,

$$\frac{1}{2} \frac{\partial^2 P(A)}{\partial A_i \partial A_j} \psi_i \psi_j + h.c. \quad (31)$$

are governed by the same couplings as the scalar interactions coming from

$$\left(\frac{\partial P(A)}{\partial A_i} \right)^2 \quad (32)$$

- Similarly, the gaugino-scalar-fermion interactions, coming from

$$-i\sqrt{2}gA_i^* T_a \psi_i \lambda^a + h.c. \quad (33)$$

are governed by the gauge couplings.

- No new couplings ! Same couplings are obtained by replacing particles by their superpartners and changing the spinorial structure.

Trilinear coupling

- A most useful example of the relation between couplings is provided by trilinear (Yukawa) couplings. To avoid complications, let's treat the abelian case:

$$P[\Phi] = h_t H U Q \quad (34)$$

where H is a Higgs superfield.

- Fermion Yukawa:

$$h_t H \psi_U \psi_Q + h.c. \quad h_t (H \bar{\psi}_R \psi_L + h.c.) \quad (35)$$

- Scalar Yukawas

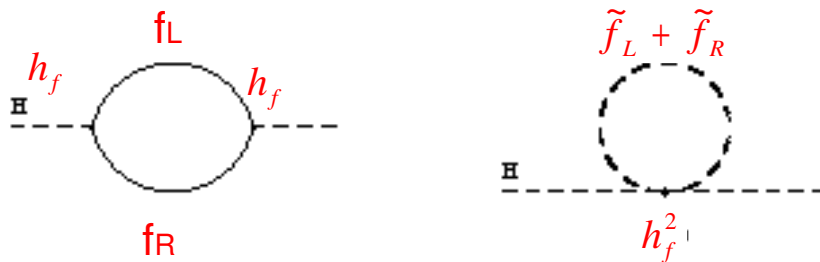
$$|h_t|^2 |H|^2 (|Q|^2 + |U|^2) \quad (36)$$

- As anticipated, same couplings of the Higgs field to fermions and to scalar fields.

Higgs Mass Parameter Corrections in SUSY

One loop corrections to the Higgs mass parameter cancel if the couplings of scalars and fermions are equal to each other

$$\delta m_H^2 = \frac{N_c h_f^2}{16\pi^2} \left[-2\Lambda^2 + 3m_f^2 \log\left(\frac{\Lambda^2}{m_f^2}\right) + 2\Lambda^2 - 2m_{\tilde{f}}^2 \log\left(\frac{\Lambda^2}{m_{\tilde{f}}^2}\right) \right]$$



If supersymmetry is exact, there is always an additional, logarithmically divergent diagram, induced by the presence of Higgs-scalar trilinear couplings, that ensure the cancellation of the logarithmic term.

Properties of supersymmetric theories

- To each complex scalar A_i (two degrees of freedom) there is a Weyl fermion ψ_i (two degrees of freedom)
- To each gauge boson V_μ^a , there is a gauge fermion (gaugino) λ^a .
- The mass eigenvalues of fermions and bosons are the same !
- Theory has only logarithmic divergences in the ultraviolet associated with wave-function and gauge-coupling constant renormalizations.
- Couplings in superpotential $P[\Phi]$ have no counterterms associated with them.
- The equality of fermion and boson couplings are essential for the cancellation of all quadratic divergences, at all orders in perturbation theory.

Supersymmetric Extension of the Standard Model

- Apart from the superpotential $P[\Phi]$, all other properties are directly determined by the gauge interactions of the theory.
- To construct the superpotential, one should remember that chiral fields contain only left-handed fields, and right-handed fields should be represented by their charge conjugates.
- SM right-handed fields are singlet under $SU(2)$. Their complex conjugates have opposite hypercharge to the standard one.
- There is one chiral superfield for each chiral fermion of the Standard Model.
- In total, there are 15 chiral fields per generation, including the six left-handed quarks, the six right-handed quarks, the two left-handed leptons and the right-handed charged leptons.

Minimal Supersymmetric Standard Model

SM particle	SUSY partner	G_{SM}
$(S = 1/2)$	$(S = 0)$	
$Q = (t, b)_L$	$(\tilde{t}, \tilde{b})_L$	$(3, 2, 1/6)$
$L = (\nu, l)_L$	$(\tilde{\nu}, \tilde{l})_L$	$(1, 2, -1/2)$
$U = (t^C)_L$	\tilde{t}_R^*	$(\bar{3}, 1, -2/3)$
$D = (b^C)_L$	\tilde{b}_R^*	$(\bar{3}, 1, 1/3)$
$E = (l^C)_L$	\tilde{l}_R^*	$(1, 1, 1)$
$(S = 1)$	$(S = 1/2)$	
B_μ	\tilde{B}	$(1, 1, 0)$
W_μ	\tilde{W}	$(1, 3, 0)$
g_μ	\tilde{g}	$(8, 1, 0)$

The Higgs problem

- Problem: What to do with the Higgs field ?
- In the Standard Model masses for the up and down (and lepton) fields are obtained with Yukawa couplings involving H and H^\dagger respectively.
- Impossible to recover this from the Yukawas derived from $P[\Phi]$, since no dependence on $\bar{\Phi}$ is admitted.
- Another problem: In the SM all anomalies cancel,

$$\begin{aligned} \sum_{quarks} Y_i &= 0; & \sum_{left} Y_i &= 0; \\ \sum_i Y_i^3 &= 0; & \sum_i Y_i &= 0 \end{aligned} \tag{37}$$

- In all these sums, whenever a right-handed field appear, its charge conjugate is considered.
- A Higgsino doublet spoils anomaly cancellation !

Solution to the problem

- Solution: Add a second doublet with opposite hypercharge.
- Anomalies cancel automatically, since the fermions of the second Higgs superfield act as the vector mirrors of the ones of the first one.
- Use the second Higgs doublet to construct masses for the down quarks and leptons.

$$P[\Phi] = h_u QUH_2 + h_d QDH_1 + h_l LEH_1 \quad (38)$$

- Once these two Higgs doublets are introduced, a mass term may be written

$$\delta P[\Phi] = \mu H_1 H_2 \quad (39)$$

- μ is only renormalized by wave functions of H_1 and H_2 .

Higgs Fields

- Two Higgs fields with opposite hypercharge.

(S = 0)

(S = 1/2)

H_1

\tilde{H}_1

(1,2,-1/2)

H_2

\tilde{H}_2

(1,2,1/2)

- It is important to observe that the quantum numbers of H_1 are exactly the same as the ones of the lepton superfield L .
- This means that one can extend the superpotential $P[\Phi]$ to contain terms that replace H_1 by L .

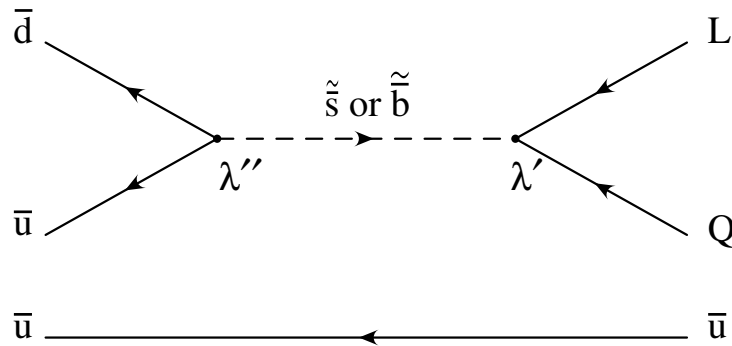
Baryon and Lepton Number Violation

- General superpotential contains, apart from the Yukawa couplings of the Higgs to lepton and quark fields, new couplings:

$$P[\Phi]_{\text{new}} = \lambda' LQD + \lambda LLE + \lambda'' UDD \quad (40)$$

- Assigning every lepton chiral (antichiral) superfield lepton number 1 (-1) and every quark chiral (antichiral) superfield baryon number 1/3 (-1/3) one obtains :
 - Interactions in $P[\Phi]$ conserve baryon and lepton number.
 - Interactions in $P[\Phi]_{\text{new}}$ violate either baryon or lepton number.
- One of the most dangerous consequences of these new interaction is to induce proton decay, unless couplings are very small and/or sfermions are very heavy.

Proton Decay



- Both lepton and baryon number violating couplings involved.
- Proton: Lightest baryon. Lighter fermions: Leptons

R-Parity

- A solution to the proton decay problem is to introduce a discrete symmetry, called R-Parity. In the language of component fields,

$$R_P = (-1)^{3B+2S+L} \quad (41)$$

- All Standard Model particles have $R_P = 1$.
- All supersymmetric partners have $R_P = -1$.
- All interactions with odd number of supersymmetric particles, like the Yukawa couplings induced by $P[\Phi]_{\text{new}}$ are forbidden.
- Supersymmetric particles should be produced in pairs.
- The lightest supersymmetric particle is stable.
- Good dark matter candidate. Missing energy at colliders.